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A power mode approach to estimating vibrational power transmitted by multiple sources

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Abstract

A power mode method for the estimation of the power transmitted to a flexible receiver by an array of point force excitations is described. The vibrational power transmitted by N discrete point forces is regarded as the power transmitted by N independent power modes following eigendecomposition of the mobility matrix of the receiving structure. Approximate expressions for the upper and lower bounds and the mean value of the transmitted power are then developed in terms of these power modes. The approach is extended to more general cases, including that where both force and moment excitations are applied to the structure and where there are velocity source excitations. Numerical examples are presented.

1. Introduction

An issue which is frequently the focus of noise and vibration control procedures is the prediction and control of the power transmitted from a resiliently mounted machine to its flexible foundation. However, the prediction of the transmitted vibration power is problematical due to the complex nature of both source and receiver [1]. For example, an exact description of the power transmitted to a flexible receiver by an array of force/velocity sources requires full knowledge of both the strength of the source excitations and the dynamic properties of the receiver. In practice, the mobilities of the source may be important. Only under ideal, limiting cases, such as are considered here, can those source mobilities be neglected. Problems then arise if there is a large number of excitation points and/or for cases where the required data are not known to sufficient accuracy.

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It is often more appropriate to approximate the main properties of the dynamic behaviour of a vibratory system, e.g., the frequency average power, rather than attempt to precisely predict the detailed response. Various methods [2–5] have been developed to approximate the power transmitted from a machine to a flexible receiver. However, there are limitations to these techniques. In this paper an alternative technique, the power mode method, is described. A set of force sources is transformed into arrays of force distributions. The transformation involves the eigenvalues and eigenvectors of the real part of the mobility matrix of the receiver. As a result, the vibrational power transmitted by N forces can be considered as being transmitted by N independent contributions, with each of them related only to one set of force distribution (eigenvector) and one eigenvalue. Thus N terms contribute to the power, rather than the N^2 terms involving the original forces. This power mode approach was first suggested in Ref. [4]. Here it is further extended and approximations are developed for the maximum and minimum possible values and the mean value of the transmitted power.

The "multipole" approach of Ref. [2] is somewhat similar, except that the transformation matrices are pre-selected "Hadamard" matrices, so that the polar mobilities can be regarded as monopole, dipole, quadrupole terms, etc. Using this approach, the power injected by many of the cross terms is often negligible. However, the receiver structure must be geometrically symmetrical or the source is a set of uncorrelated outputs.

There are two main advantages to the power mode approach. Firstly, it allows expressions for the upper and lower bounds and the mean value of the transmitted power to be developed in a simple manner. Secondly, this approach can be used for cases where both force and moment excitations are involved.

In the next section the power mode theory is developed for an array of point forces applied to a region of a structure whose properties are uniform and homogeneous. Then various approximations are developed. Following this, more general situations are considered. These include the case of combined force and moment sources and that of velocity source excitations. Finally some numerical examples are presented.

2. Power mode theory

Suppose an array of N time harmonic forces is applied to a structure at a frequency ω. The time-averaged power transmitted to the receiver can be expressed as

$$P = \tfrac{1}{2}\mathrm{Re}\{\mathbf{F}^{\mathrm{H}}\mathbf{V}\}, \quad \mathbf{V} = \bar{\mathbf{M}}\mathbf{F}, \tag{1}$$

where $\mathbf{F}$ is the vector of amplitudes of the forces, $\mathbf{V}$ is the vector of amplitudes of the velocities of the receiving structure at the excitation points, $\bar{\mathbf{M}}$ is the complex mobility matrix of the receiver structure, and the superscript H denotes the conjugate transpose. Eq. (1) can be written as

$$P = \tfrac{1}{4}\mathrm{Re}\{\mathbf{F}^{\mathrm{H}}\mathbf{V} + \mathbf{V}^{\mathrm{H}}\mathbf{F}\} = \tfrac{1}{4}\mathrm{Re}\{\mathbf{F}^{\mathrm{H}}(\bar{\mathbf{M}} + \bar{\mathbf{M}}^{\mathrm{H}})\mathbf{F}\}. \tag{2}$$

Since $(\bar{\mathbf{M}} + \bar{\mathbf{M}}^{\mathrm{H}}) = 2\,\mathrm{Re}\{\bar{\mathbf{M}}\}$, the transmitted power becomes

$$P = \tfrac{1}{2}\mathbf{F}^{\mathrm{H}}\mathbf{M}\mathbf{F}, \tag{3}$$

where $\mathbf{M}$ is the real part of the complex mobility matrix $\bar{\mathbf{M}}$. Eq. (3) shows that the power transmission depends only on the real part of the mobility matrix, whose imaginary part can therefore be ignored as far as power transmission is concerned.

Since Eq. (3) is in a non-negative definite quadratic form, $\mathbf{M}$ is a real, symmetric and non-negative definite matrix. By matrix theories [6,7], $\mathbf{M}$ can be decomposed into the form

$$\mathbf{M} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{\mathrm{T}}, \tag{4}$$

where $\mathbf{\Lambda}$ is a real and non-negative diagonal matrix of the eigenvalues λ_n of $\mathbf{M}$, $\mathbf{\Psi}$ is the orthogonal matrix composed of the corresponding eigenvectors (in columns), so that $\mathbf{\Psi}\mathbf{\Psi}^{\mathrm{T}} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{\Psi} = \mathbf{I}$, and the superscript T denotes the transpose. The eigenvalues are arranged in descending order, i.e.,

$$\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_N \geqslant 0. \tag{5}$$

The eigenvalues satisfy the following relations [7]:

$$\sum_{n=1}^{N} \lambda_n = \sum_{n=1}^{N} M_{nn}, \tag{6}$$

$$\sum_{n=1}^{N} \lambda_n^2 = \sum_{m=1}^{N}\sum_{n=1}^{N} M_{mn}^2 = \|M\|_2, \tag{7}$$

where $\|M\|_2$ is the second order norm of matrix $\mathbf{M}$. From Eqs. (6) and (7), the mean value of λ_n and its standard deviation σ are found to be

$$\bar{\lambda} = \sum_{n=1}^{N} M_{nn}/N, \tag{8}$$

$$\sigma = \sqrt{\frac{\|M\|_2}{N} - \frac{1}{N^2}\left(\sum\nolimits_{n=1}^{N} M_{nn}\right)^2}. \tag{9}$$

Let the force vector $\mathbf{F}$ now be weighted by $\mathbf{\Psi}$ so as to give a new set of forces defined by

$$\mathbf{Q} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{F}. \tag{10}$$

It follows that

$$\sum_{n=1}^{N} |Q_n|^2 = \sum_{n=1}^{N} |F_n|^2. \tag{11}$$

Combining Eqs. (3), (4) and (10), the power transmitted to the receiver can then be re-written as

$$P = \frac{1}{2}\mathbf{Q}^{\mathrm{H}}\mathbf{\Lambda}\mathbf{Q} = \frac{1}{2}\sum_{n=1}^{N} |Q_n|^2 \lambda_n. \tag{12}$$

Eq. (12) shows that the vibrational power transmitted to the receiver by N forces can be regarded as the power transmitted by N independent contributions, each of them related only to one set of force distribution (eigenvector) and one eigenvalue. In Ref. [8], "radiation modes" have been used to describe the power radiated by a vibrating surface into a surrounding acoustic medium, in which the sound power radiation from a set of velocity distribution (radiation mode) on the structure is independent of the amplitudes of the other velocity distributions. Therefore, by analogy to the "radiation modes", Eq. (12) may be defined as a set of independent "power modes". The force vector **Q** may thus be called power mode force vector, which is given in terms of **F** and the eigenvectors of the real part of the mobility matrix, and the eigenvalue λ_n called power mode mobilities. Eq. (12) is in contrast to Eq. (3) where the power is given by sum of N^2 terms involving the physical forces **F**.

Conceptually, Eq. (12) is useful. However, it does not provide any practical advantages over Eq. (3) since full knowledge of **M** is required to determine its eigenproperties. However, advantages do occur because simple approximations can be developed for the transmitted power based on power mode theory. These are developed in the next section.

3. Power transmission approximation based on the power mode approach

As mentioned in Section 1, it is often more appropriate to approximate the main properties of the transmitted power rather than attempt to predict precisely the detailed response, especially if the properties of the receiver structure are not known exactly. Therefore in this section the power mode theory is used to find simple approximations of the transmitted power. These give estimates of upper and lower bounds for the transmitted power, as well as its mean value. The actual power thus lies in some range between these upper and lower bounds.

3.1. Upper and lower bounds of the transmitted power

Expressions for the upper and lower bounds of the transmitted power can be derived from power mode theory. Combining Eqs. (5), (11), and (12), strict upper and lower bounds are given by

$$P_{up} = \frac{1}{2}\left(\sum_{n=1}^{N} |F_n|^2\right)\lambda_1, \tag{13}$$

$$P_{low} = \frac{1}{2}\left(\sum_{n=1}^{N} |F_n|^2\right)\lambda_N. \tag{14}$$

It is seen that the bounds on power depend only on the maximum and minimum power mode mobilities of the receiver structure λ_1 and λ_N as well as the magnitudes of the force sources $|F_n|$, regardless of the distribution and the relative phases of the force sources [4]. Therefore, it is a good simplification to estimate the transmitted power using upper and lower bounds. Since the usefulness of such an approximate approach depends on the width of the range formed by the upper and lower limits, Eqs. (13) and (14) are very suitable for cases where the maximum and minimum power mode mobilities of the receiver structures are comparable.

Generally, the correlation between the individual excitation points of the receiver becomes of decreasing importance as the wavelength of the structure decreases. Therefore when the wavelength of the receiver structure is very short, it is reasonable to neglect the correlations between the individual excitations, at least when frequency averaged, so that each individual excitation is taken to be independent. If it is also assumed that the local driving point properties of the receiver have the same order of magnitude, the eigenvalues λ_n typically are then of the same order of magnitude. As a result, both λ_1 and λ_N will often lie within, say, one standard deviation of the mean $\bar{\lambda}$, so that they can be simply approximated, using Eqs. (8) and (9), as

$$\lambda_1 \approx (\bar{\lambda} + \sigma), \tag{15}$$

$$\lambda_N \approx (\bar{\lambda} - \sigma). \tag{16}$$

As a result, Eqs. (13) and (14) can be re-written as

$$P_{up} \approx \frac{1}{2}\left(\sum_{n=1}^{N} |F_n|^2\right)(\bar{\lambda} + \sigma), \tag{17}$$

$$P_{low} \approx \frac{1}{2}\left(\sum_{n=1}^{N} |F_n|^2\right)(\bar{\lambda} - \sigma). \tag{18}$$

So far the upper and lower bounds for the power transmitted to a short-wavelength structure can be approximated using Eqs. (17) and (18). In such cases, many or all power modes contribute significantly to the transmitted power rather than just a few of them. In particular, when the receiver structure is very flexible (e.g., such that $kl \gg 1$ where k is the wavenumber and l the distance between the excitation points), so that each forcing point can be taken as uncorrelated with the others, the power mode mobilities and the power mode forces can be simply approximated by

$$\lambda_n \approx M_{nn}, \tag{19}$$

$$Q_n \approx F_n. \tag{20}$$

Eq. (12) then becomes

$$P \approx \sum_{n=1}^{N} \left(\frac{1}{2}|Q_n|^2 \lambda_n\right) \approx \sum_{n=1}^{N} \left(\frac{1}{2}|F_n|^2 M_{nn}\right). \tag{21}$$

The above expression indicates that each power mode contributes significantly to the total power transmission at high frequencies.

In other cases, however, e.g., if the wavelength is very long, individual excitations may be strongly correlated. For example, when the response is dominated by a single resonant mode, then at the resonant frequency, the driving point mobility tends to be quite close to the transfer mobility [2], i.e.,

$$\mathrm{O}[M_{nn}] \approx \mathrm{O}[M_{mn}], \quad m \neq n. \tag{22}$$

Here it is still assumed that the local driving point properties of the receiver have same orders of magnitude. As a result, the maximum power-mode mobility λ_1 can be much larger than the minimum one λ_N, which implies that only a few of the lower power modes give significant power transmission. Then the power range formed by Eqs. (13) and (14) will be too broad to be of practical value, due to a very small lower bound. Under such circumstances, it is more useful to replace the lower bound by the approximation for the power associated with only the first power mode which plays a dominant role in the total power transmission, while the upper bound is still approximated using Eq. (17).

Eq. (22) implies that the eigenvector $\boldsymbol{\psi}_1$ has the approximate form

$$\boldsymbol{\psi}_1 = \frac{1}{\sqrt{N}}[1 \;\; 1 \;\; \cdots \;\; 1]^{\mathrm{T}}. \tag{23}$$

When λ_1 is still approximated using Eq. (15), the power transmitted by the first power mode can then be approximated as

$$P_1 \approx \frac{1}{2N}\left|\sum_{n=1}^{N} F_n\right|^2 (\bar{\lambda} + \sigma). \tag{24}$$

Thus the power transmitted to a long-wavelength receiver can be estimated using the range formed by Eqs. (17) and (24).

Especially in the very low-frequency range where $kl \ll l$, all forces are in effect applied at same point. It then follows that

$$\lambda_1 \approx \sum_{n=1}^{N} M_{nn}, \quad \lambda_{2,3,\ldots,N} \approx 0, \tag{25}$$

$$|Q_1|^2 \approx \frac{1}{N}|F_1 + F_2 + \cdots + F_N|^2. \tag{26}$$

Eq. (12) then becomes

$$P \approx \tfrac{1}{2}|Q_1|^2 \lambda_1 \approx \frac{1}{2}\left(\left|\sum_{n=1}^{N} F_n\right|^2\right) \sum_{n=1}^{N} M_{nn}/N. \tag{27}$$

Eq. (27) shows that in the low-frequency range the power can be regarded as being transmitted by the first order power mode only, i.e., one power mode dominates. This is similar to the monopole in the multipole approach.

3.2. Mean value of the transmitted power

In the above subsection, upper and lower bounds of the transmitted power were found which define a fairly narrow range within which the transmitted power lies. It is also useful to estimate the mean value of the power over a range of frequencies. An estimate of the mean value of the transmitted power can be found by taking the average over all the power modes.

The mean square power mode force can be found from Eq. (11) to be

$$E[|Q_n|^2] = \frac{1}{N}\sum_{n=1}^{N} |F_n|^2. \tag{28}$$

The mean power modal mobility is given by Eq. (7). The mean value of the transmitted power, when averaged over all the power modes, can thus be approximated in terms of the mean square force and the mean point mobility as

$$E[P] = \frac{N}{2}\left(\frac{1}{N}\sum_{n=1}^{N}|F_n|^2\right)\left(\frac{1}{N}\sum_{n=1}^{N}M_{nn}\right). \tag{29}$$

Eq. (29) is in a very simple form, being equivalent to approximating the mean of a product by the product of their means, and gives an estimate of the frequency average of the transmitted power. This result was also given in Ref. [3].

Thus the power transmitted to a receiver structure by an array of point forces can be described in terms of upper and lower bounds and a mean value.

4. Combined force and moment excitations

The translational motion normal to the surface of the seating is usually the dominant mechanism of power transmission from a machine source to a flexible supporting structure [3]. However, it is known that in many cases of practical interest, vibration sources apply moments as well as forces. The power transmitted by moment excitations is generally greater at higher frequencies [9,10]. Therefore it is necessary to consider also moment excitations.

Suppose the source array $\mathbf{F}$ is formed partly by a set of forces and partly by a set of moments. The real part of the mobility matrix $\mathbf{M}$ of the receiver structure is now composed of force and moment point mobilities and transfer mobilities, and the transmitted power can be written in the same form as Eq. (3). However, since both $\mathbf{F}$ and $\mathbf{M}$ consist of elements with different units, the approximations developed in Section 3 are no longer applicable. In this section, a scaling technique to deal with this problem is described. The main principle of this scaling technique is to scale the mobility matrix $\mathbf{M}$ by a specified diagonal matrix to give a new "dimensionless" matrix, and then to weight the physical force vector using the same diagonal matrix to give a new set of forces with the same units. As a result, the power mode approach described in the previous sections can then be applied.

Let $\mathbf{M}$ be scaled by such a real diagonal matrix $\mathbf{D}_C$ defined as

$$D_{C,nn} = \frac{1}{\sqrt{M_{nn}}}, \tag{30}$$

where $D_{C,nn}$ and M_{nn} are the nth diagonal elements of $\mathbf{D}_C$ and $\mathbf{M}$, respectively, so that

$$\mathbf{M}_C = \mathbf{D}_C\mathbf{M}\mathbf{D}_C. \tag{31}$$

Let $\mathbf{F}$ be weighted by $\mathbf{D}_C^{-1}$ so as to give a new set of forces defined by

$$\mathbf{F}_C = \mathbf{D}_C^{-1}\mathbf{F}. \tag{32}$$

Combining Eqs. (31) and (32) with Eq. (3) gives

$$P = \tfrac{1}{2}\mathbf{F}_C^{\mathrm{H}}\mathbf{M}_C\mathbf{F}_C. \tag{33}$$

Since the scaled mobility matrix $\mathbf{M}_C$ is real, symmetric, non-negative and dimensionless and $\mathbf{F}_C$ is a vector of the scaled forces with the same units, Eq. (33) then meets all the requirements of the power mode theory.

$\mathbf{M}_C$ is decomposed into the form

$$\mathbf{M}_C = \mathbf{\Psi}_C \mathbf{\Lambda}_C \mathbf{\Psi}_C^{\mathrm{T}}, \tag{34}$$

where $\mathbf{\Lambda}_C$ is the real and non-negative diagonal matrix of the eigenvalues of $\mathbf{M}_C$, and $\mathbf{\Psi}_C$ is the orthogonal matrix composed of the corresponding eigenvectors (in columns). Let the scaled force vector $\mathbf{F}_C$ be weighted by $\mathbf{\Psi}_C$. A new set of scaled power mode forces can then be written as

$$\mathbf{Q}_C = \mathbf{\Psi}_C^{\mathrm{T}} \mathbf{F}_C = \mathbf{\Psi}_C^{\mathrm{T}} \mathbf{D}_C^{-1} \mathbf{F}. \tag{35}$$

Consequently, Eq. (33) can be written in terms of the power modes as

$$P = \tfrac{1}{2} \mathbf{Q}_C^{\mathrm{H}} \mathbf{\Lambda}_C \mathbf{Q}_C = \frac{1}{2} \sum_{n=1}^{N} |Q_{C,n}|^2 \lambda_{C,n}, \tag{36}$$

where $\lambda_{C,n}$ is the nth diagonal element of $\mathbf{\Lambda}_C$.

As a result, approximate expressions for the upper and lower bounds as well as the mean value are then developed by analogy with Eqs. (17), (18) (or Eq. (24)) and Eq. (29), with $\mathbf{F}$ being replaced by $\mathbf{F}_C$ and M by $\mathbf{M}_C$.

The above scaling technique in effect scales the individual excitations by a factor equal to the square root of the real part of the input mobility so that the elements of $\mathbf{F}_C$ have the same units. An alternative scaling procedure can be used, with the scaling matrix $D_{C,n}$ being given by

$$D_{C,nn} = \frac{1}{\sqrt{M_{nn}^{\infty}}}, \tag{37}$$

where M_{nn}^{∞} is the characteristic point mobility of the receiver structure, i.e., the point mobility if the receiver structure is extended to infinity. Comparing Eq. (30) with Eq. (37), it can be expected that the former gives better estimates of the transmitted power while the latter allows for uncertainties in the properties of the receiver structure, e.g., the boundary conditions.

Since the scaling matrices described in Eqs. (30) and (37) are all frequency-dependent, different scaling matrices may have different influences on the performance of the power approximations.

4.1. Other cases

Similar results can be obtained for velocity/rotational velocity excitations in the same manner as that described above for force/moment sources. In this case, however, the mobility matrix of the receiver structure is replaced by the corresponding impedance matrix, with the roles of the force and velocity vectors being reversed.

In some cases there may also be force excitations which act in different directions on the receiver (e.g., in-plane and out-of-plane forces), so that the corresponding input mobilities M_{nn} may have different orders of magnitude—in-plane motion is usually much stiffer than out-of-plane motion, for example. If such excitations input significant power, then this situation can be treated using the same scaling approach described above.

5. Numerical examples

Numerical examples are considered in this section. The system model chosen is a thin rectangular plate with four simply supported edges. The plate has a length of 2 m in the x direction, a width of 0.9 m in the y direction and a thickness of 0.005 m. The chosen material of the plate is perspex with a Young's modulus of $4.4 \times 10^9\,\mathrm{N/m^2}$, a density of $1152\,\mathrm{kg/m^3}$, a material loss factor of 0.05 and the Poisson ratio of 0.38. The plate is first assumed to be excited by point forces, and then by co-located force/moment excitations. A running frequency average has been taken over a frequency band of width three times the mean modal spacing, to illustrate the broader features of the power transmission. The exact results are found using Eq. (3), i.e., the classical mobility matrix method.

5.1. Plate with force excitations

First the plate is assumed to be excited by three-point forces $F_1 = 1$, $F_2 = 2e^{j\pi/3}$ and $F_3 = 0.5e^{-j\pi/4}$, located at $(x_1, y_1) = (0.37, 0.45)\,\mathrm{m}$, $(x_2, y_2) = (0.89, 0.45)\,\mathrm{m}$ and $(x_3, y_3) = (1.34, 0.45)\,\mathrm{m}$, respectively. Since here there are three-point forces, there are consequently three power modes.

Using Eq. (4) a receiver structure can be characterized by a set of power mode mobilities. Fig. 1 shows the three power mode mobilities of the plate as a function of frequency. The first power mode tends to be much larger than the others at lower frequencies, but becomes comparable to the other power mode mobilities as frequency increases. This implies that the transmitted power is dominated by the first power mode at lower frequencies while more power modes give significant contributions at higher frequencies.

This trend is further illustrated in Fig. 2, which shows the power transmitted by each power mode together with the total power. It can be seen that the power transmitted by the first power mode dominates the total power transmission for the low modal overlap area (e.g., below 70 Hz where the modal overlap factor is less than unity), but the significance of the other two power

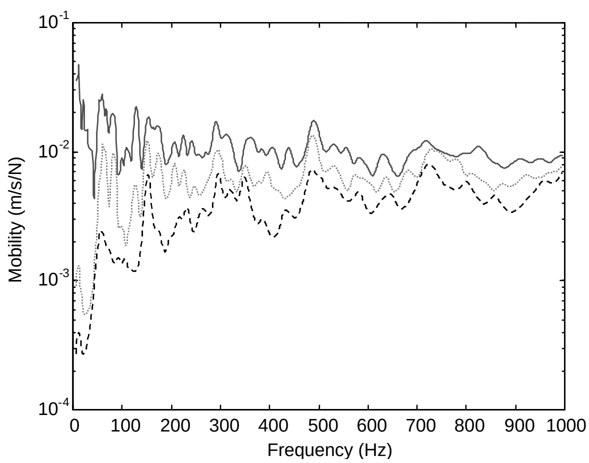

Fig. 1. Power mode mobilities of the plate: first order (——), second order (......), and the third order (– – –).

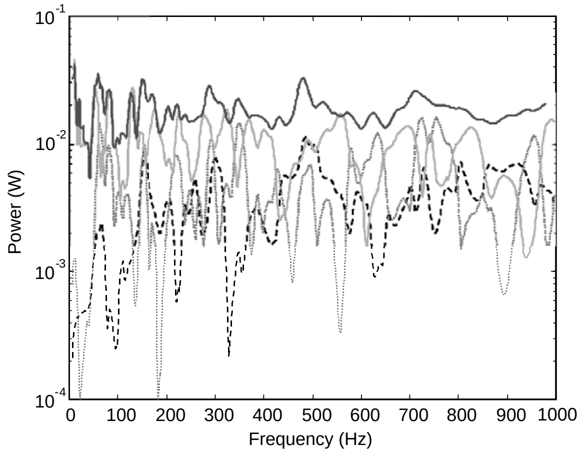

Fig. 2. Total power transmission and the power transmitted by each power mode: total power transmission (——, Eq. (3)); power transmitted from the first (——), second (......) and third (– – –) power modes (Eq. (12)).

modes increases as the modal overlap of the receiver increases. Therefore, it is quite reasonable to estimate the lower power bound using the first power mode approximation for a stiff and low modal overlap receiver. It should be noted that the power associated with an individual power mode depends on both the power mode mobility and the corresponding power mode force, the latter being determined by not only the excitation forces themselves, but also the forcing positions, as given in Eq. (10). Thus there may be a small power mode mobility and a large power mode force, or vice versa. As a result, the lower order power modes do not necessarily transmit more power than the higher order ones, as shown in Fig. 2.

In Section 3, approximations for the upper and lower bounds and the mean of the transmitted power were developed. Fig. 3 shows the power transmitted to the plate together with these approximations. It can be seen that the mean power expression in Eq. (29) gives a fairly good approximation to the transmitted power. The upper and the lower bounds expressed by Eqs. (17) and (18) are very useful approximations for the transmitted power, provided the modal overlap factor of the receiver structure is high enough (e.g., more than 3 (above 200 Hz) in Fig. 3). Below this frequency the lower bound is more accurately approximated by the power transmitted by the first power mode, given in Eq. (24).

5.2. Simultaneous force/moment excitations

The power mode approach can be applied to cases where both force and moment excitations are involved using the scaling technique described in Section 4. This is investigated here by assuming the point force sources comprise not only the forces of the previous example, but also consist of co-located moment excitations $M_{x1} = 0.05\,\mathrm{N\,m}$, $M_{x2} = 0.075\mathrm{e}^{\mathrm{j}2\pi/3}\,\mathrm{N\,m}$ and $M_{x3} = 0.05\mathrm{j}\,\mathrm{N\,m}$. Fig. 4 shows the power transmitted to the plate and the approximations when the scaling matrix of Eq. (30) is used. It is seen that the bounds provide a narrow range for the power and the mean value is a good approximation.

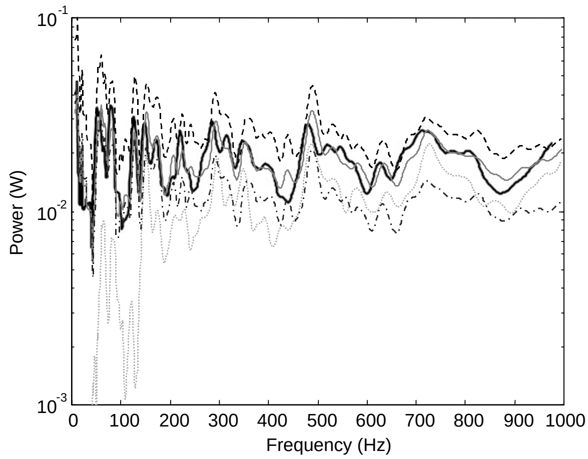

Fig. 3. Exact and approximate power transmissions to the plate when the plate is excited by three point forces: exact value (——, Eq. (3)); approximations for the mean (——, Eq. (29)), upper bound (–––, Eq. (17)), lower bound (......, Eq. (18)), and first power mode (–·–·–·, Eq. (24)).

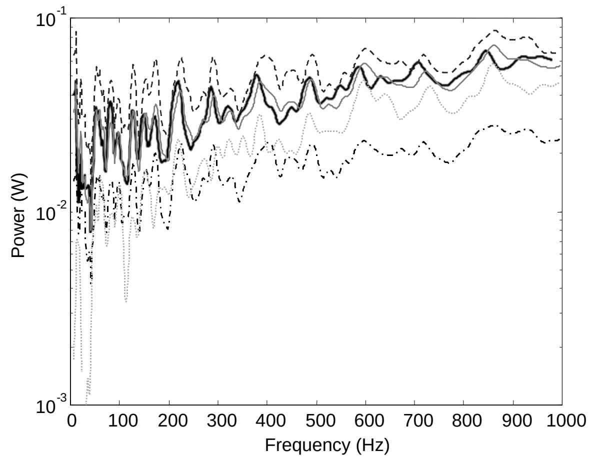

Fig. 4. Exact and approximate power transmissions when the plate is excited by co-located force/moment excitations (scaled by Eq. (30)): exact value (——, Eq. (3)); approximations for the mean (——, Eq. (29)), upper bound (––– , Eq. (17)), lower bound (......, Eq. (18)), and first power mode (–·–·–·, Eq. (24)).

Fig. 5 shows the power transmitted to the plate and the approximations when the scaling matrix of Eq. (37) is used. It is seen that the expressions plotted in Fig. 4 give better estimates of the transmitted power than Fig. 5, as would be expected. This is because the scaling approach for Fig. 4 needs exact information of the point mobilities M_{nn} of the receiver, but the scaling approach for Fig. 5 only needs the characteristic point mobility terms M_{nn}^{∞} of the receiver. However, the latter scaling approach can be more useful when the receiver structures have some uncertainties, e.g. boundary condition uncertainties.

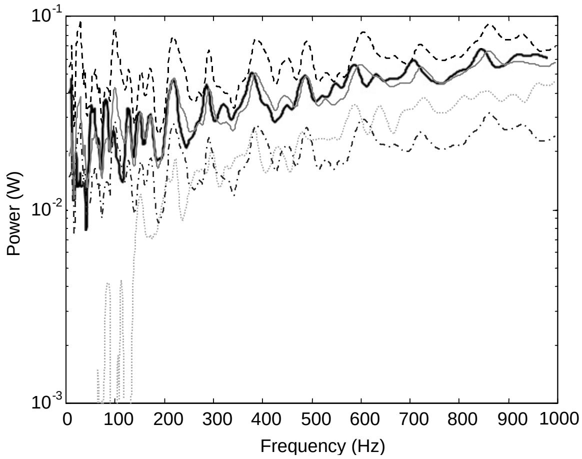

Fig. 5. Exact and approximate power transmissions when the plate is excited by co-located force/moment excitations (scaled by Eq. (37)): exact value (—— , Eq. (3)); approximations for the mean (——, Eq. (29)), upper bound (– – –, Eq. (17)); lower bound (......, Eq. (18)), and first power mode (– · – · – , Eq. (24)).

6. Concluding remarks

In this paper a power mode method for estimating the power transmitted to a flexible receiver by an array of point force excitations was described. Based on power mode theory, the vibrational power transmitted by N discrete point forces was regarded as the power transmitted by N independent power modes following eigendecomposition of the real part of the mobility matrix of the receiving structure. Simple expressions were developed for approximating the upper and lower bounds and the mean value of the transmitted power in terms of these power modes. It also has been shown that these approximations can be extended to more general cases, including that where both force and moment excitations are applied to the structure and where there are velocity source excitations. Finally, numerical results were presented for the case of a plate excited at a number of points.

This power mode technique is currently being further developed to approximate the vibration power transmission between a stiff source and a flexible receiver through discrete couplings.

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